EE479 Digital Control Systems

Assignment # 1
Read Ch. 10.1 –10.5
Submit the following problems:

10.3 (a), (b), (c), (d), (e)

(a) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

(b) $K = [0.4 \quad 0.2]$

(c) $K = [0.8 \quad 0.2]$

(d) for (b) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ **for (c)** $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(e) Use "placepol " function

10.4 (a), (b), (c)

(a) $\dot{x} = -0.05x + u, \qquad y = x$

(b) $k = 0.05$

(c) Obtain the closed-loop block diagram representation

10.7 (a), (b), (c)

(a) $G = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

(b) $G_{ec}(s) = \dfrac{3.2s + 6.4}{s^2 + 10s + 32}$

(c) Find $1 + G_{ec}(s)G_p(s)$

$$C.E. = s^4 + 12s^3 + 52s^2 + 96s + 64 = 0$$
$$= (s^2 + 4s + 4)(s^2 + 8s + 16) = 0$$

10.8 (a), (b), (c)

(a) $G = 0.25$

$\dot{\hat{x}} = -0.3\hat{x} + u + 0.25y, \qquad u = -0.05\hat{x}$

(b) $G_{ec}(s) = \dfrac{0.0125}{s + 0.35}$

(c) Find $1 + G_{ec}(s)G_p(s)$

$$C.E. = s^2 + 0.4s + 0.03 = 0$$
$$= (s + 0.1)(s + 0.3) = 0$$

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Assignment # 2
Read Ch. 11.1 –11.6
Submit the following problems:

11.2 (a), (b), (c)

(a) $e(t) = 2e^{-2.107t}$

(b) Use the real-transformation time delay $z^{-1}E(z) = \mathcal{Z}[x(kT-T)]$

$e(t) = 2e^{-2.107(t-0.05)}u(t-0.05)$

(c) Verify by finding the z-transform of each function

11.5 (a), (b), (c), (d), (e), (f)

(a) Use long division $E(z) = 3z^{-2} + 7.2z^{-3} - \cdots$

(b) $e(k) = 60 + 15(0.5)^k - 75(0.9)^k$

(c) Use dimpulse(num, den, k)

(d) Verify, (yes)

(e) Final value theorem give the correct answer since $E(z)$ has a simple pole at z=1.

(f) Find $e(\infty)$ from (b) and the final value theorem, $e(\infty) = 60$

11.6 (a), (d), (e)

(a) Power series $E(z) = z^{-1} + 1.8z^{-2} + 2.44z^{-3} + \cdots$

$e(0) = 0, \quad e(1) = 1, \quad e(2) = 1.8, \quad e(3) = 2.44, \quad e(4) = 2.952$

Partial fraction expansion $e(k) = 5 - 5(0.8)^k$, Check

(d) Power series $E(z) = z^{-3} + 1.8z^{-4} + 2.44z^{-5} + \cdots$

$e(0) = 0, \quad e(1) = 0, \quad e(2) = 0, \quad e(3) = 1, \quad e(4) = 1.8, \quad e(5) = 2.44$

Partial fraction expansion $e(k) = 5 - 7.8125(0.8)^k$ for $k \geq 2$, Check

(e) In MATLAB, use xk=impulse(num, den,k) for k =5 to verify the power series also use the command [r, p, k] = residue(num1, den1) to verify PFE of $\frac{E(z)}{z}$

11.8 (a), (b), (c)

(a) $x(2) = -2, \quad x(3) = 0, \quad x(4) = 2, \quad x(5) = -2, \quad x(6) = 0, \quad x(7) = 2$

(b) $x(k) = 2.309\sin(k120)$, check for $x(k)$

(c) Write a MATLAB program and verify

11.9 (a), (b), (c), (d), (e)

(a) $x(0) = 1, \quad x(1) = 4, \quad x(2) = 11, \quad x(3) = 26, \quad x(4) = 57$

(b) Modify MATLAB program in Example 11.6 and verify the results in (a)

(c) $X(z) = \dfrac{z^3}{(z-1)^2(z-2)}$

(d) $X(z) = 1 + 4z^{-1} + 11z^{-2} + 26z^{-3} + 57z^{-4} + \cdots$

(e) $x(k) = -3 - k + 4(2)^k$ check and verify the results in (a)

11.11 (a), (b), (c), (d)

(a) $e(k) = 0.1(1.2)^k$, final value unbounded

(b) $e(k) = 0.1(0.8)^k$, $e(\infty) = 0$

(c) $e(k) = 0.333[1 - (0.7)^k]$, $e(\infty) = 0.333$, apply the final value theorem and verify

(d) $e(k)$ is sinusoidal (find the inverse z-transform) $e(\infty)$ does not exist.

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Assignment # 3
Read Ch. 11.6 –11.9
Submit the following problems:

11.15 (a), (b)

(a) Write the difference equations for simulation diagram in Figure 11.15 (a) and Figure 11.15(b)

(b) Obtain the transfer function for each simulation diagram

Figure 11.15 (a) $\dfrac{M(z)}{E(z)} = \dfrac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0}$

Figure 11.15 (b) $\dfrac{M(z)}{E(z)} = \dfrac{\beta_2 z^2 + \beta_1 z + \beta_0}{z^2 + \alpha_1 z + \alpha_0}$

$\Rightarrow \quad \alpha_i = a_i, \quad \beta_i = b_i$

11.16 (b)

(b) $m(k) = 1.8m(b-1) - 0.9m(k-2) + 0.333e(k) - 0.556e(k-1) + 0.3e(k-2)$

11.19 (a), (b), (c), (d), (e), (f)

(a) Draw the simulation diagram

(b)
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.6 & 1.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 0.03 & 0.035 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(c) Use $\dfrac{Y(z)}{U(z)} = C[zI - A]^{-1} B,$ and show $\dfrac{Y(z)}{U(z)} = \dfrac{0.035z + 0.03}{z^2 - 1.2z + 0.6}$

(d) Obtain directly from the difference equation and verify the result in (c)

(e) Apply Mason’s rule and verify the result in (c)

(f) Use the function [Gnum, Gden]= ss2tf(A, B, C, D); G=tf(Gnum, Gden, -1)

11.21 (b), (f)

(b) $y(k) = 3.17 - 9.29(0.3)^k + 6.11(0.1)^k$

(f) [Gnum, Gden]= ss2tf(A, B, C, D); G=tf(Gnum, Gden, -1)

11.23 (a), (b), (c)

(a) $$\phi(k) = \begin{bmatrix} -\frac{1}{2}k2^k + 2^k & -\frac{1}{2}k2^k \\ \frac{1}{2}k2^k & \frac{1}{2}k2^k + 2^k \end{bmatrix}$$

(b) $x(1) = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \quad x(2) = \begin{bmatrix} 8 \\ -12 \end{bmatrix}, \quad x(3) = \begin{bmatrix} 20 \\ -28 \end{bmatrix},$

(c) $x(3) = \begin{bmatrix} 20 \\ -28 \end{bmatrix}$

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Assignment # 4
Read Ch. 12.1 –12.9
Submit the following problems:

12.1 (a), (d)

(a) $E^*(s) = \dfrac{e^{Ts}}{e^{Ts} - e^{-3T}}$

(d) $E^*(s) = \dfrac{\frac{3}{2}e^{Ts}}{e^{Ts} - 1} - \dfrac{\frac{1}{2}e^{Ts}}{e^{Ts} - e^{-2T}}$

11.11 (a), (b), (c)

(a) $c(kT) = e^{-kT} = e^{-0.1k}$

(b) $c(t) = e^{-t}$

(c) No effect, the sampler zero-order hold rebuilds a constant signal exactly

12.13 (a), (b)

(a) $G(s) = \dfrac{1 - e^{Ts}}{s} G_p(s)$, find $G(z) = \dfrac{0.1813}{z - 0.8187}$

$A(s) = E(s)\dfrac{1}{s+1} = \dfrac{1}{s(s+1)}$, find $A(z) = \dfrac{0.09516z}{(z-1)(z-0.9048)}$

$C(z) = A(z)G(z) = \dfrac{0.01725z}{(z-1)(z-0.9048)(z-0.8187)}$

$c(kT) = 1 = 1.904(0.9048)^k + 0.9047(0.8187)^k$

(b) Plot $c(kT)$, simulate the system with SIMULINK and verify the plot

12.15 (a), (b), (c)
Correction: Change the given difference equation to $m(k) = m(k-1) + 0.5e(k)$

(a) $\dfrac{C(z)}{E(z)} = \dfrac{0.1z}{z^2 - 2z + 1}$

(b) Draw the simulation diagram

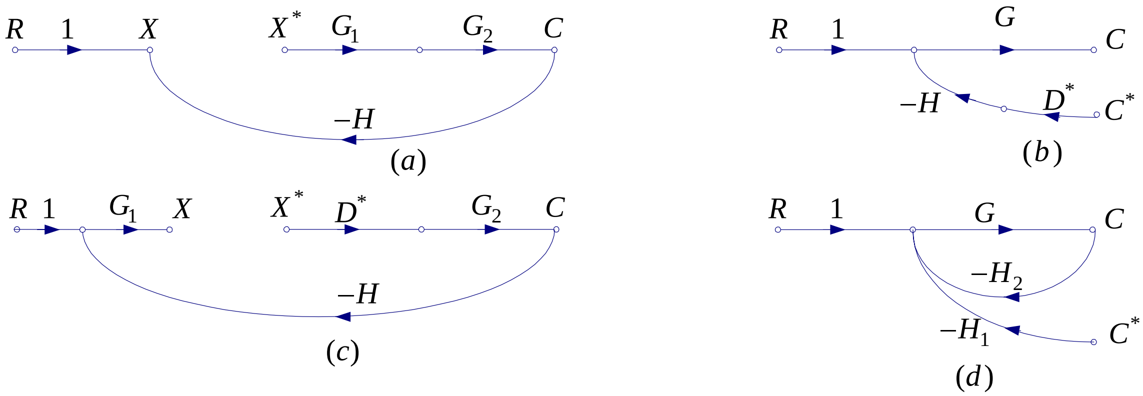
(c)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 0.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

12.16 (a), (b), (c), (d)

Follow the procedure 1-5 outlined in Chapter 12 (Page 506). Remember if there is a sampler between $G_1(s)$ and $G_2(s)$, in the sampled block diagram the gain is $G_1(z)G_2(z)$. If there no sampler between $G_1(s)$ and $G_2(s)$, we have a single transfer function denoted by $\overline{G_1G_2}(z)$.



Determine $\frac{C(z)}{R(z)}$ for each block diagram.

12.17 (a), (b), (c), d

(a) $\mathrm{G(z)} = \frac{0.05(z+1)}{z^2 - 2z + 1}$

(b) $\mathrm{T(z)} = \frac{0.05k(z+1)}{z^2 - (z - 0.05k)z + (1 + 0.05k)}$

(c)

$\mathrm{D(z)} = \frac{2.2z - 1}{z}$

$\mathrm{T(z)} = \frac{0.05k(2.2z^2 + 1.2z - 1)}{z^3 - (z - 0.11k)z^2 + (1 + 0.06k)z - 0.05k}$

(d) Define num, den, use Gs = tf(num, den), Gz = c2d(Gs, 0.1 ‘zho’)

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Assignment # 5
Read Ch. 13
Submit the following problems:

13.1 (a) through (f) Correction in Figure P13.1 The plant transfer function is

$$G_p(s) = \frac{K}{s+2}$$

(a) $T = 1$ sec $\quad T(z) = \frac{0.432}{z + 0.297} \quad c(k) = 0.333[1 - (-0.297)^k]$

(b) $T = 0.1$ sec $\quad T(z) = \frac{0.0906}{z - 0.728} \quad c(k) = 0.333[1 - (0.728)^k]$

(c) $\quad T(s) = \frac{1}{s+3} \quad c(t) = 0.333[1 - e^{-3t}]$

(d) Use MATLAB to plot the above responses.

```
Ta=1;  ta=0:1:5;
Tb=0.1; tb=0:0.1:5;
tc =0:0.01:5;
cka = 0.333*(1-(-0.297).^ta/Ta);
stairs(ta, cka, 'k'), grid on
hold on                % Hold current Graph
        Similarly continue for response (b)
        ⋮
        Define the response for (c) and use plot function
        ⋮
```

(e) Comment

(f) Use **c2d,** and **feedback** functions to obtain the sample-data transfer functions and use **ltiview** to obtain the three responses on the same graph.

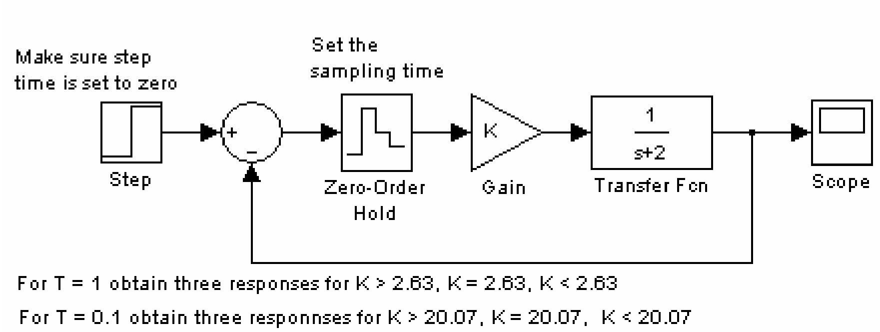
13.2 (a), (b), (c)

$T = 1$ sec $\quad K < 2.63$

(a) $T = 0.1$ sec $\quad K < 20.07$

Continuous $\quad 0 < K < \infty$

Include the Simulink diagram and the responses for



Comment, is the continuous-time system stable for all $0 < K < \infty$?

(b) For $K = 8$, T=0.255 sec

(c) Obtain the Simulink responses for $K = 8$, T=0.27, T =0.255, T = 0.2, and comment.

13.6 (a) Use Bilinear transformation $z = \frac{1+W}{1-W}$ and Routh array instead of Jury test

(a) $0 < K < 5$

13.8 (a), (b), (c), (d), (e), (f)

(a) $G(z) = \frac{0.4261(z+0.8467)}{(z-1)(z-0.6065)}$ Plot the z-plane root locus , Find j-w axis intersect, breakaway and reentry points.
(b) Assume RL intersect with unit circle is at 0.564+j0.825, Find $K = 1.1$
(c) $K = 0.0552$ for double roots at $z = 0.791$
(d) Plot root locus using MATLAB and verify (a)
(e) Verify (b)
(f) Verify (c) by plotting the step response in MATLAB

13.13 (a), (b), (c)

Use Bilinear transformation $z = \frac{1+W}{1-W}$ and Routh array instead of Jury test

(a) $0 < K < 1.096$
(b) For $K = 1.096$ find roots of the C.E.

13.16 (a), (b), (c), (d)

(a) $T = 1$ sec, $G(z) = \frac{0.4323}{z-0.1353}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$
(b) $T = 0.1$ sec, $G(z) = \frac{0.09063}{z-0.8187}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$
(c) $G(s) = \frac{1}{s+2}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$
(d) See problem 13.1(e) and comment.