Review Problems EE-303 Test # 1
(Hadi Saadat)

CHAPTER 1

Signal Types

- Continuous-time signal (CT) Analog
- Discrete-time signal (DT) Sampled
- Quantized signal
- Digital signal

Signal Symmetry

A signal $x(t)$ is even if $x(-t) = x(t)$. A signal $x(t)$ is odd if $x(-t) = -x(t)$. The even and odd parts of a signal are given by

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

- The sum of two even functions is even.
- The sum of two odd functions is odd.
- The sum of an even function and an odd function is neither even or odd
- The product of two even functions is even.
- The product of two odd functions is even.
- The product of an even function and an odd function is odd.

1. Find the even and odd components of each of the following signals

(a) $x(t) = \cos t + \sin t + \sin t \cos t$

(b) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

(c) $x(t) = 1 + t\cos t + t^2 \sin t$

(d) $x(t) = \left(1 + t^3\right)\cos^3(10t)$

Periodic and aperiodic signals

A continuous-time signal is periodic if and only if $x(t+T) = x(t)$ for all t. where

$T = T_0,\ 2T_0,\ 3T_0,\ \ldots$, and T_0 is the fundamental period. ($T_0 = \frac{1}{f_0}$).

Any other signal not satisfying the above condition is called *aperiodic* signal. The sum of two or more continuous-time periodic waves is periodic if the ratio of each pair of individual frequencies is a rational fraction, i.e., their frequencies are commensurable.

Problem (1-11 text)

2. The sinusoidal signal

$$x(t) = 3\cos(200t + \frac{\pi}{6})$$

is passed through a square-law device defined by the input-output relation

$$y(t) = x^2(t)$$

Show that the output $y(t)$ consists of a dc component and a sinusoidal component.

(a) Specify the dc component

(b) Specify the amplitude and fundamental frequency of the sinusoidal component in the output $y(t)$.

Phasor signals and Spectra

A complex signal is defined as

$$x(t) = Ae^{j(\omega_0 t+\theta)} = \overrightarrow{X}e^{j\omega_0 t}$$

where $\overrightarrow{X} = Ae^{i\theta} = A\angle\theta$ is a rotating phasor. The sinusoidal signal $x(t) = A\cos(\omega_0 t+\theta)$ may be written as the real part of the rotating phasor, i.e., $x(t) = \Re\left\{\overrightarrow{X}e^{j\omega_0 t}\right\}$. Alternatively, $x(t)$ may be written as one-half of the counter rotating signals, i.e.,

$$x(t) = \frac{1}{2}\left[x(t) + x^*(t)\right]$$

An alternative way to visualize the sinusoidal signal $x(t)$ in the frequency domain is in the form of two plots. One the amplitude A as the function of frequency f, and the other its phase angle θ as a function of f. These plots are referred to as *single-sided spectra*. If the amplitude and phase angle plots are made for the oppositely rotating phasors we obtain the so-called *double-sided spectra*.

Problems (1-12, 1-43 text)

Singularity functions, signal description, sketching signals, and operation on signals

Examples of singularity functions are: unit impulse $\delta(t)$, unit step $u(t)$, and unit ramp $r(t)$.

$$r(t) = tu(t) \qquad u(t) = \frac{d}{dt}r(t) \qquad \delta(t) = \frac{d}{dt}u(t)$$

More complex functions can be built up of sums or products of singularity functions.

An impulse signal is mathematically described by

$$\delta(t) = 0 \text{ for } t \neq 0 \text{ and } \int_{-\infty}^{\infty}\delta(t)dt = 1$$

An important property of an impulse signal is the sifting property

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0) = x(t_0)$$

Another commonly used signal is the pulse signal $x(t) = \Pi(t)$, defined as

$$\Pi(t) = \begin{cases} 1 & -\frac{1}{2} \leq \text{t} \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

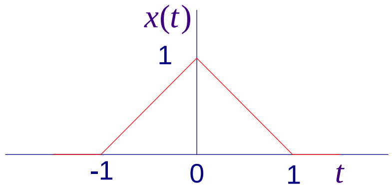
Time Shift: If $x(t)$ is a continuous function, the time-shifted signal is defined as $y(t) = x(t-t_0)$. If $t_0 > 0$, the signal is shifted to the right, and if $t_0 < 0$, the signal is shifted to the left.

Time Reversal: If $x(t)$ is a continuous function, the time-reversed signal is defined as $y(t) = x(-t)$.

Time Scaling: If $x(t)$ is a continuous function, a time-scale version of this signal is defined as $y(t) = x(at)$. If $a > 1$, the signal $y(t)$ is a compressed version of $x(t)$, i.e., the time interval is compressed to $\frac{1}{a}$. If $0 < a < 1$, the signal $y(t)$ is a stretched version of $x(t)$, i.e., the time interval is stretched by $\frac{1}{a}$. When operating on signals, the time-shifting operation must be performed first, and then the time-scaling operation is performed.

Problems (1-14, 1-18, 1-22, 1-26 text)

3. A triangular pulse signal $x(t)$ is depicted below.



Sketch each of the following signals:

(a) $x(3t)$

(b) $x(3t+2)$

(c) $x(-2t-1)$

(d) $x(0.5t-1)$

Energy and power signals

The energy of a CT signal $x(t)$ is given by

$$E = \lim_{T\to\infty}\int_{-T}^{T} |x^2(t)|\,dt$$

The power of a CT signal $x(t)$ is given by

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} |x^2(t)|\,dt \quad \text{or} \quad P = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x^2(t)|\,dt$$

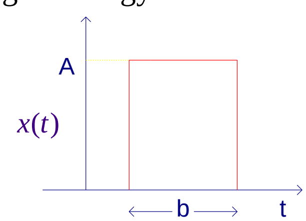
For periodic signals $P = \frac{1}{T}\int_{0}^{T} |x^2(t)|\,dt \;= \dfrac{\text{Energy in one period}}{\text{T}}$ and $X_{rms} = \sqrt{P}$

For discrete-time signals

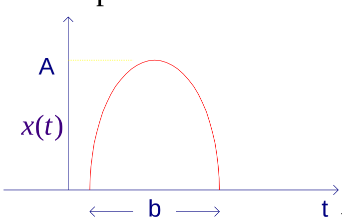
$$E = \sum_{n=-\infty}^{\infty} |x^2[n]| \qquad \text{P} = \lim_{N\to\infty}\frac{1}{N}\sum_{-N}^{N} |x^2[n]| \qquad \text{For periodic DT signal } P = \frac{1}{N}\sum_{0}^{N-1} |x^2[n]|$$

Energy signal is a signal with a finite energy and zero power. A power signal is a signal with nonzero power and infinite energy. All periodic signals are power signals.

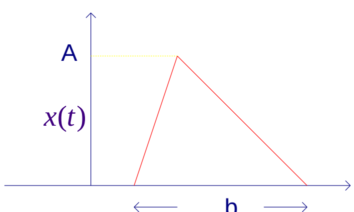
Signal energy for three useful pulse shapes are



Rectangular pulse $E = A^2 b$



Half-cycle sinusoid pulse $E = \frac{1}{2}A^2 b$



Triangular pulse $E = \frac{1}{3}A^2 b$

Problems (1-33, 1-34, 1-38 text)

Power Spectral Density

Denoting the energy spectral density of a signal $x(t)$ by $G(f)$, the signal energy is

$$E = \int_{-\infty}^{\infty} G(f)df$$

Denoting the power spectral density of a signal $x(t)$ by $S(f)$, the average power of the signal is

$$P = \int_{-\infty}^{\infty} S(f)df$$

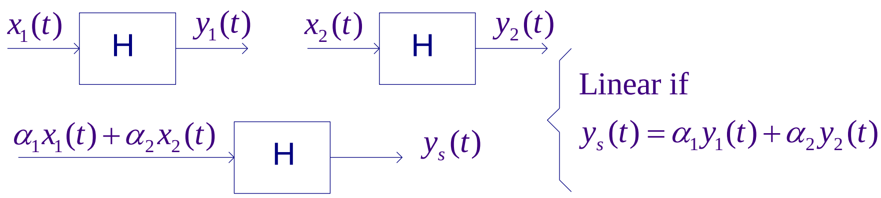
Problems (1-44, 1-45 text)

CHAPTER 2

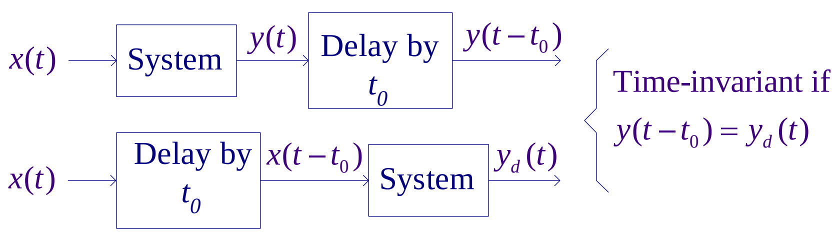
System Classification

Linearity: A linear system is one for which superposition applies and implies three constraints:

1. The system equation must involve only linear operators.
2. The system must contain no internal independent sources.
3. The system must be relaxed (with no zero initial conditions).



Time-invariant: A system is time-invariant (fixed) if its input-output relationship does not change with time. When one or more coefficients are function of time, the system is time-varying.



Causal (non-anticipating) A system is causal if its present response depends on the present or past values of the input signal.

Static (instantaneous or memoryless): A system is static if the output $y(t)$ depends only on the instantaneous value of the input $x(t)$. The system equation contains no derivative, and every term in x and y has identical arguments.

Dynamic (system with memory): A dynamic system is characterized by differential equations. Its present response depends on both present and past inputs. A system is dynamic if energy storage is present.

Problems (2-5, 2-6, 2-7, 2-11 text)

The Convolution integral

The term convolution means "folding." Convolution is an invaluable tool to the engineer because it provides a means of viewing and characterizing physical systems. Knowing the system impulse response $h(t)$, the system response $y(t)$ to an excitation $x(t)$ is expressed as

$$y(t) = x(t) * h(t) \quad \Rightarrow \quad y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

or

$$y(t) = h(t) * x(t) \quad \Rightarrow \quad y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

Steps to evaluate the convolution integral:

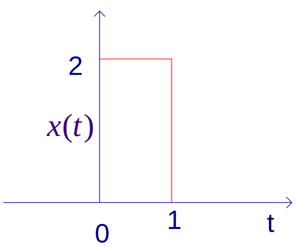
1. Express $x(t)$, and $h(t)$ by intervals and find the convolution range using the *pairwise sum*.
2. Folding – Take the mirror image of $h(t)$ about the ordinate axis to obtain $h(-\lambda)$.
3. Displacement – shift or delay $h(-\lambda)$ by t to obtain $h(t-\lambda)$.
4. For each range, locate $h(t-\lambda)$ relative to $x(\lambda)$. Keep the end points of $h(t-\lambda)$ in terms of t
5. For each range, integrate $h(t-\lambda)\, x(t)$ over their overlap duration to find the convolution.

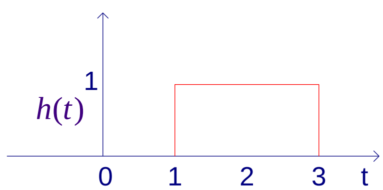
The above process can be done by switching the roles of input and the impulse response, i.e., folding and shifting $x(t)$.

Problem (2.17 text)

4. Find the convolution of the signals $x(t)$, and $h(t)$ sketched below:

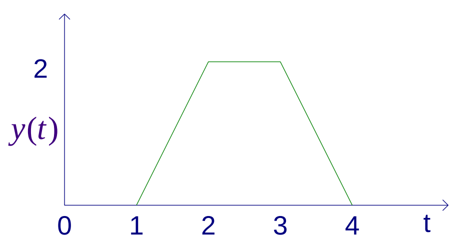
(a)



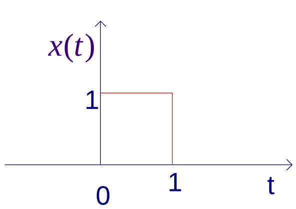


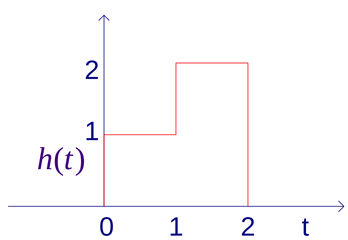
Ans.

$$y(t) = \begin{cases} 0 & 0 \le t \le 1 \\ 2t-2 & 1 \le t \le 2 \\ 2 & 2 \le t \le 3 \\ 8-2t & 3 \le t \le 4 \\ 0 & t \ge 4 \end{cases}$$



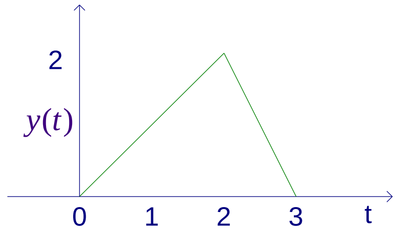
(b)



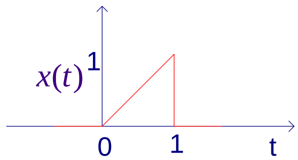


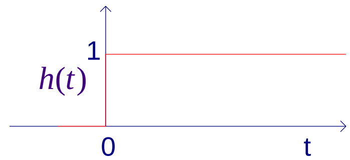
Ans.

$$y(t)=\begin{cases} \text{t} & 0\le t\le 1 \\ \text{t} & 1\le t\le 2 \\ \text{6-2t} & 2\le t\le 3 \\ 0 & \text{otherwise} \end{cases}$$



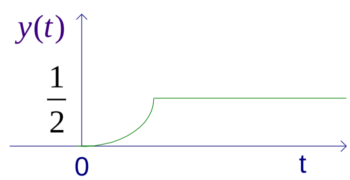
(c)



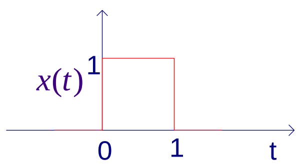


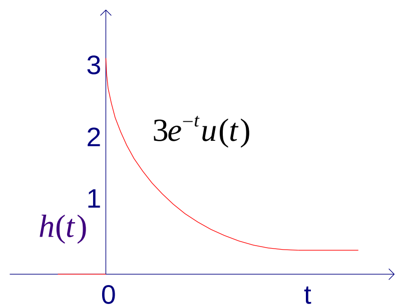
Ans.

$$y(t)=\begin{cases} \frac{1}{2}\text{t}^2 & 0\le t\le 1 \\ \frac{1}{2} & t\ge 1 \end{cases}$$



(d)





Ans.

$$y(t)=\begin{cases} 3(1-e^{-t}) & 0\le t\le 1 \\ 3(\text{e - 1})\text{e}^{-t} & t\ge 1 \end{cases}$$

5. Find analytical convolution for the following signals:

(a) $x(t)=e^{-2t}u(t)$ and $h(t)=e^{-t}u(t)$

Ans. $e^{-t}-e^{-2t} \qquad t\ge 0$

(b) $x(t)=e^{-\alpha t}u(t+3)$ and $h(t)=e^{-t}u(t-1)$

Ans. $(t+2)e^{-\alpha t}u(t+2)$

(c) $x(t)=u(t+1)-u(t-1)$ and $h(t)=u(t+1)-u(t-1)$

Ans. $r(t+2)-2r(t)+r(t-2)$

(d) $x(t)=u(t+3)-u(t-1)$ and $h(t)=u(t+1)-u(t-1)$

Ans. $r(t+4)-r(t+2)-r(t)+r(t+2)$

Impulse Response

The impulse response $h(t)$ of the first-order system described by

$$\frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t) \quad \text{with} \quad y(0) = 0$$

is equivalent to solving the homogeneous equation

$$\frac{dy(t)}{dt} + a_0 y(t) = 0 \quad \text{with} \quad y(0) = b_0 \qquad \Rightarrow y(t) \ = \ b_0 e^{-a_0 t}$$

Similarly, the impulse response $h(t)$ of a second order system

$$\frac{d^2 y(t)}{dt} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

with zero initial conditions can be found as the solution to the homogeneous equation

$$\frac{d^2 y(t)}{dt} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = 0 \quad \text{with } y(0) = 0 \,\&\, \frac{dy(0)}{dt} = b_0$$

The classical solution of the homogenous second-order differential equation is obtained according to the roots of the characteristic equation $s^2 + a_1 s + a_0 = 0$

Problems (2-22, 2-23)

6. Find the impulse response of the circuits shown.